Chapter 1

Basics of Geometry

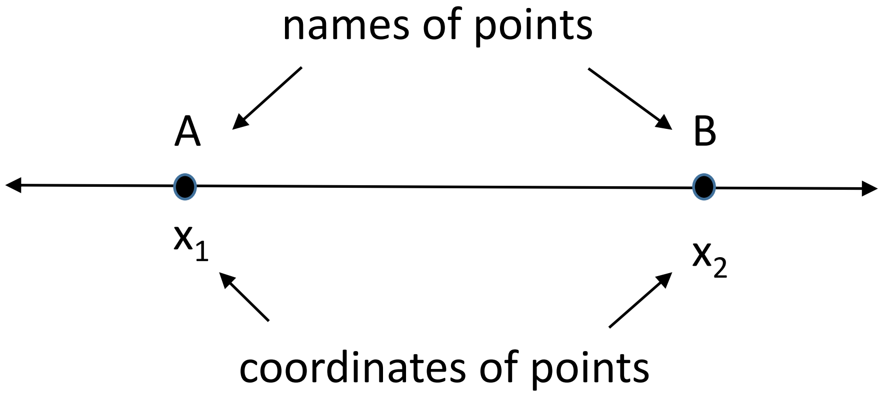
Section 3
Segments and Their Measures

GOAL 1: Using Segment Postulates

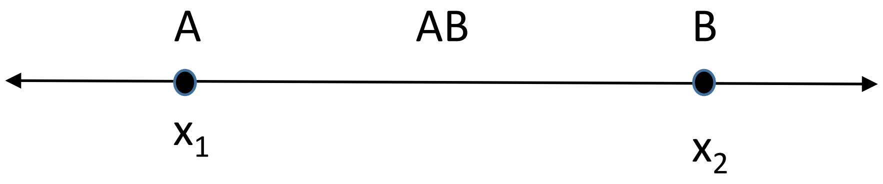
In geometry, rules that are accepted without proof are called __postulates_ or _axioms_. Rules that are proved are called __theorems__.

POSTULATE 1: Ruler Postulate

The points on a line can be matched one to one with the real numbers. The real number that corresponds to a point is the ___coordinate__ of the point.



The __distance__ between points A and B, written as AB, is the absolute value of the difference between the coordinates of A and B.

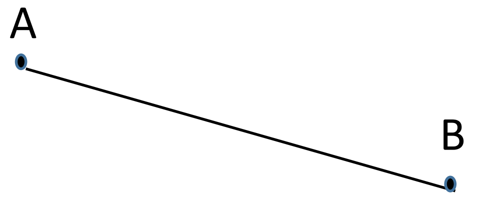


$AB = |x_2 - x_1|$

AB is also called the _length__ of $\overline{AB}$.

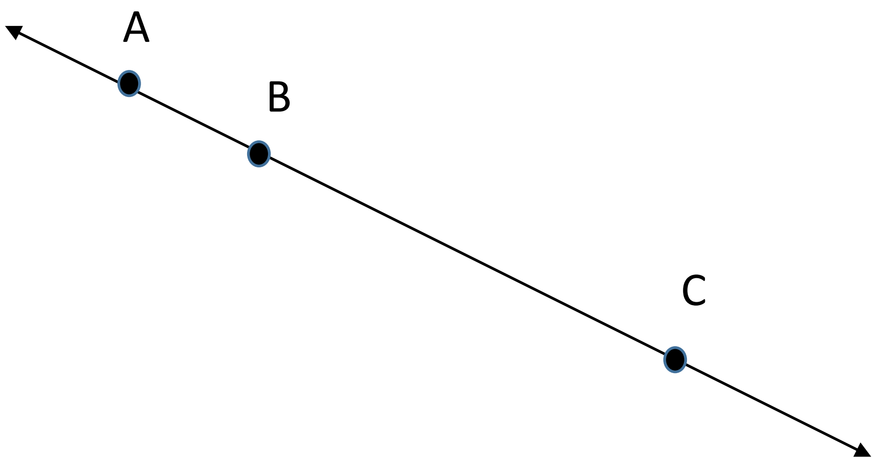
Example 1: Finding the Distance Between Two Points

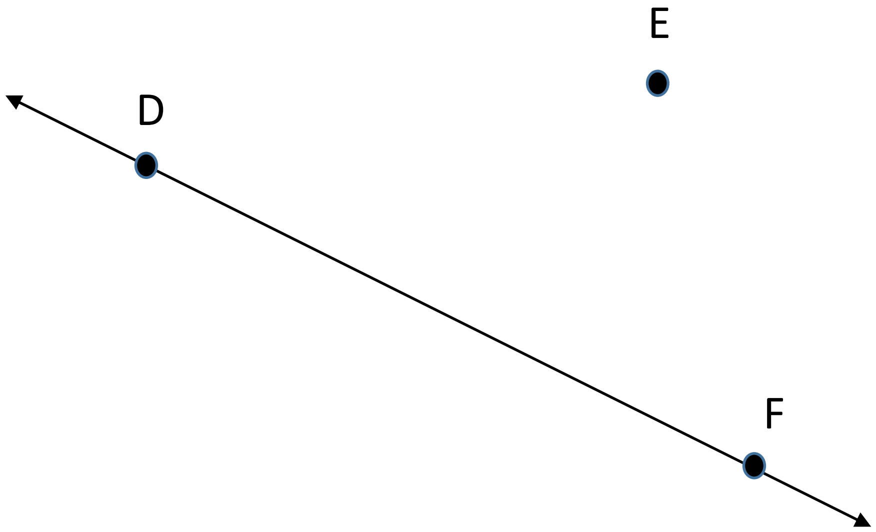
Measure the length of the segment to the nearest millimeter.



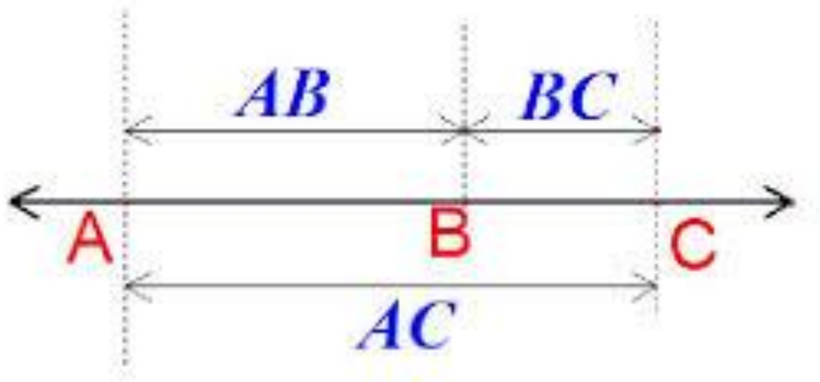
2.5 cm

When three points lie on a line, you can say that one of them is _between_ the other two. This concept applies to collinear points only. For instance, in the figures below, point _B_ is between points _A_ and _C_, but point _E_ is not between points _D_ and _F_.





POSTULATE 2: Segment Addition Postulate



If B is between A and C, then _AB_ + _BC_ = _AC_.

If _AB_ + _BC_ = _AC_, then B is between A and C.

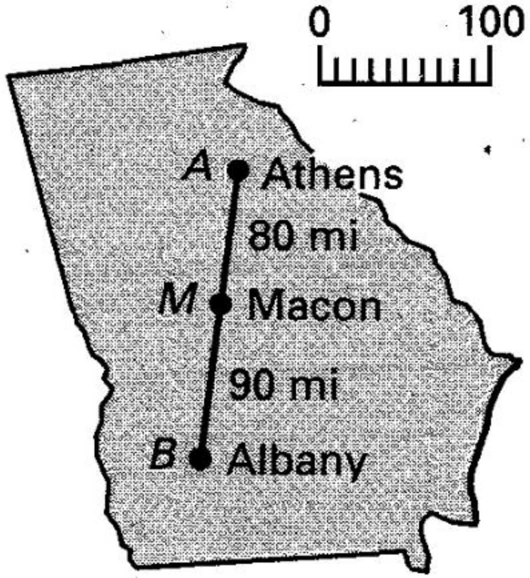
Example 2: Finding Distance on a Map

Use the map to find the distances between the three cities that lie on a line.

AM + MB = AB ← Segment Addition Postulate

80 + 90 = 170

170 miles

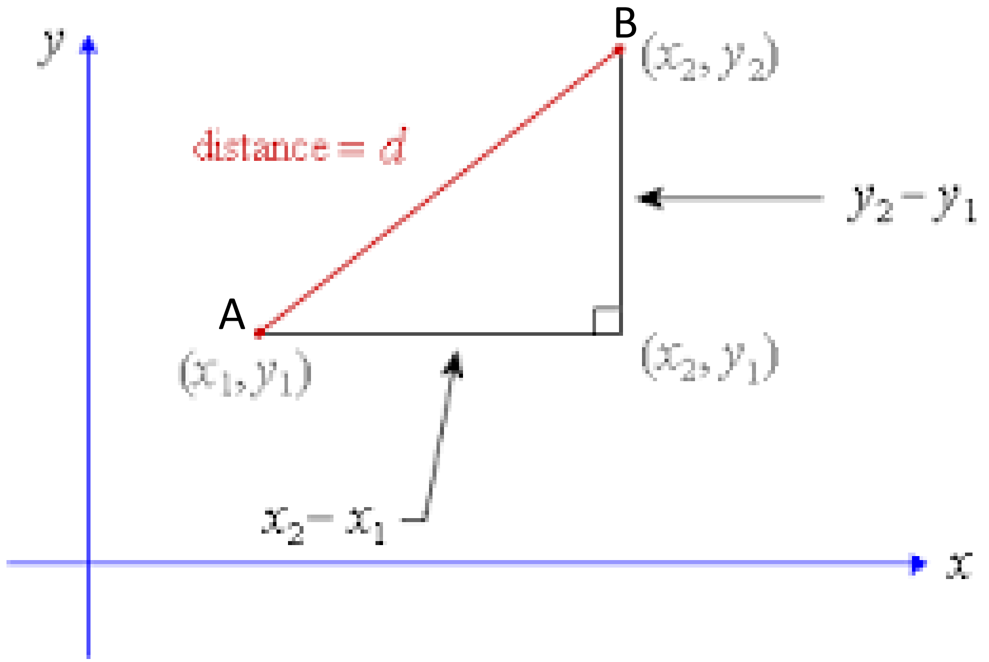


GOAL 2: Using the Distance Formula

The Distance Formula is used for computing the distance between two points in a coordinate plane.

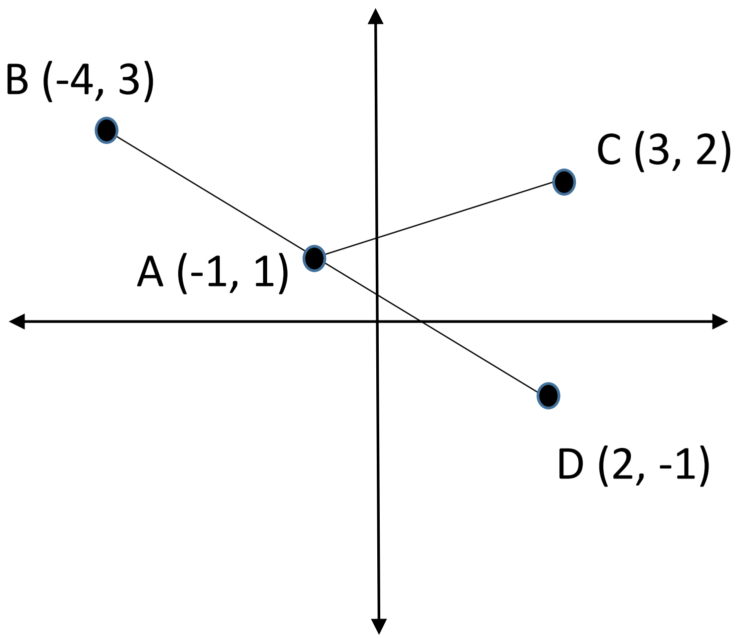
If A(x_1, y_1) and B(x_2, y_2) are points in a coordinate plane, then the distance between A and B is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Example 3: Using the Distance Formula

Find the lengths of the segments. Tell whether any of the segments have the same length.



AB (-1, 1) (-4, 3)

$\sqrt{(-1+{}^{+}4)^2+(1-3)^2}$

$\sqrt{3^2+(-2)^2} \rightarrow \sqrt{9+4} \rightarrow \sqrt{13}$

AC (-1, 1) (3, 2)

$\sqrt{(-1-3)^2+(1-2)^2}$

$\sqrt{(-4)^2+(-1)^2} \rightarrow \sqrt{16+1} \rightarrow \sqrt{17}$

AD (-1, 1) (2, -1)

$\sqrt{(-1-2)^2+(1+{}^{+}1)^2}$

$\sqrt{(-3)^2+(2)^2} \rightarrow \sqrt{9+4} \rightarrow \sqrt{13}$

Segments that have the same length are called ___congruent segments___.

There is a special symbol, ≅ , for indicating congruence.

IMPORTANT NOTE:

Lengths are equal.

$AB = AD$

Segments are congruent.

$\overline{AB} \cong \overline{AD}$

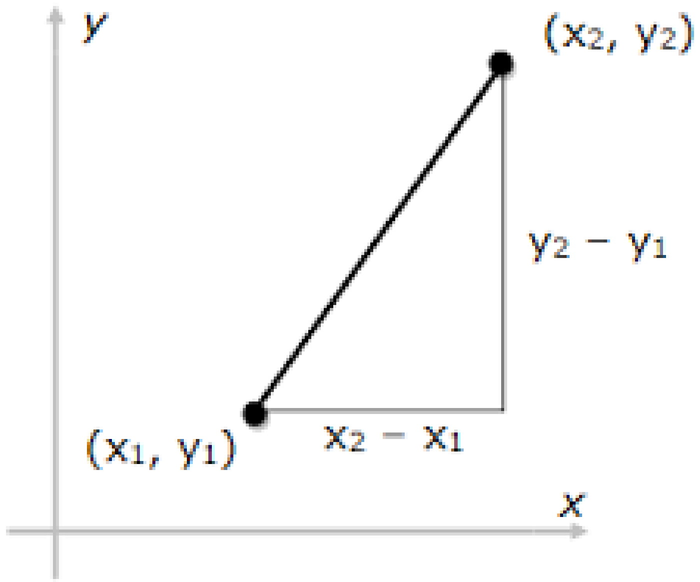
The Distance Formula is based on the Pythagorean Theorem, which we will see again when we work with right triangles in Chapter 9.

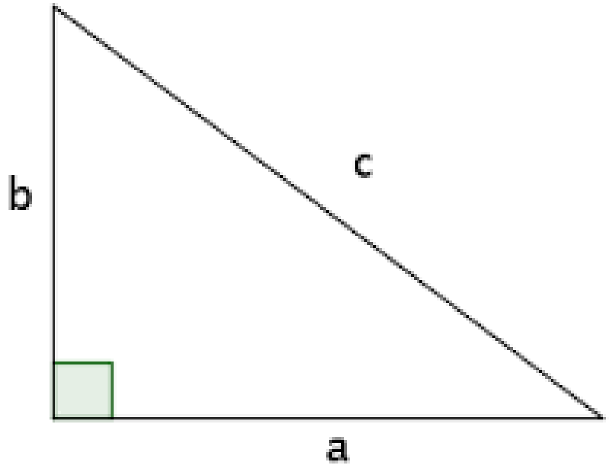
Distance Formula

$$(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Pythagorean Theorem

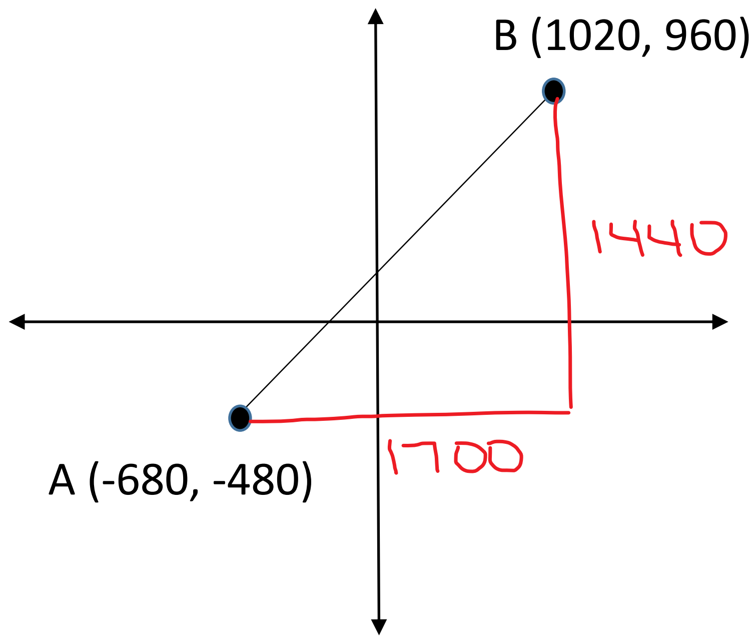
$$c^2 = a^2 + b^2$$





Example 4: Finding Distances on a City Map

On the map, the city blocks are 340 feet apart east-west and 480 feet apart north-south.



a. Find the walking distance between A and B.

1700 + 1440 = 3140 feet

a. What would the distance be if a diagonal street existed between the two points?

$\sqrt{(1020 - -680)^2 + (960 - -480)^2}$

$\sqrt{1700^2 + 1440^2}$

$\sqrt{4963600} \Rightarrow \approx 2228$ ft